



# SENSITIVITY ANALYSIS – THEORY AND PBPK APPLICATIONS

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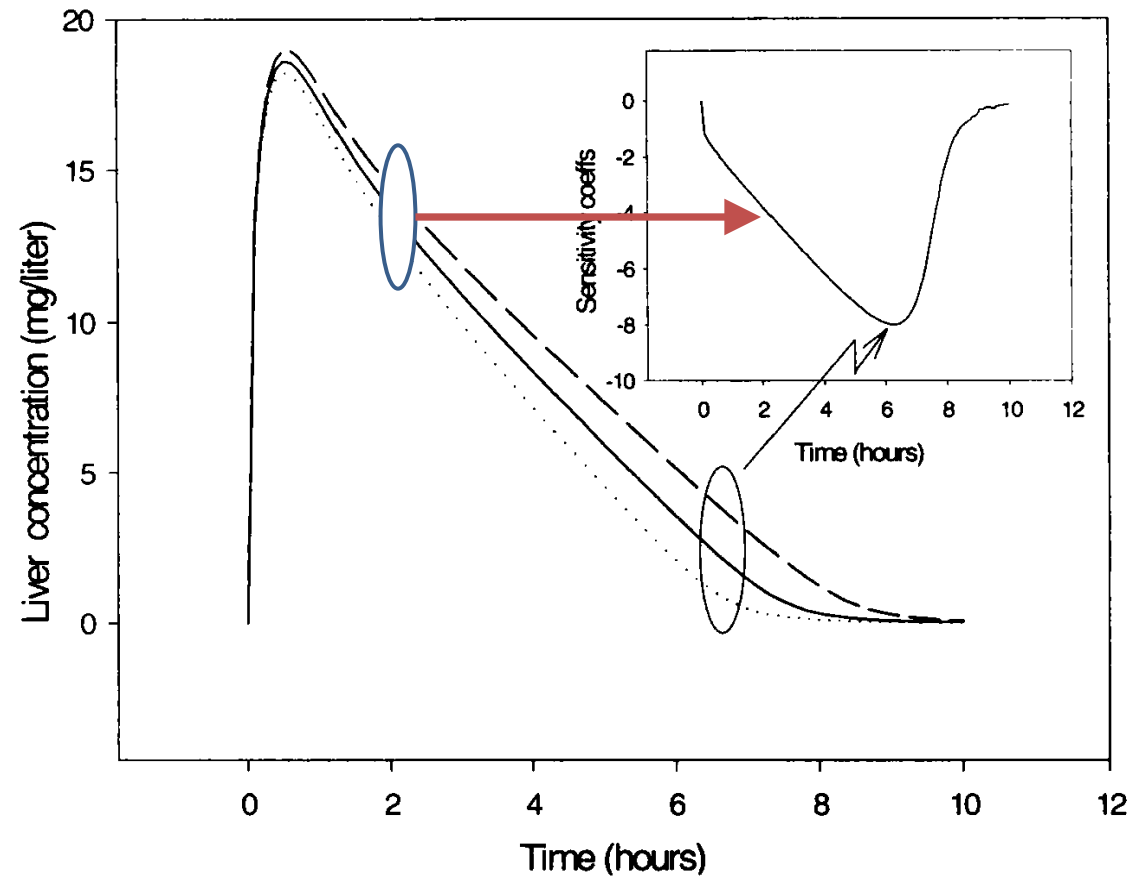
# Sensitivity Analysis

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- Mathematical application – estimates the impact a parameter change has on a model output over time.
  - How much change in output occurs due to change in a parameter.
  - Each parameter will have a related sensitivity coefficient.
  - Application: slopes from gas uptake chambers are related to ability to estimate  $V_{max}$  and  $K_m$ .
  - Different concentrations have different slopes and groups of data are needed for a single estimate.
  - Because of different slopes we can get unique estimates.

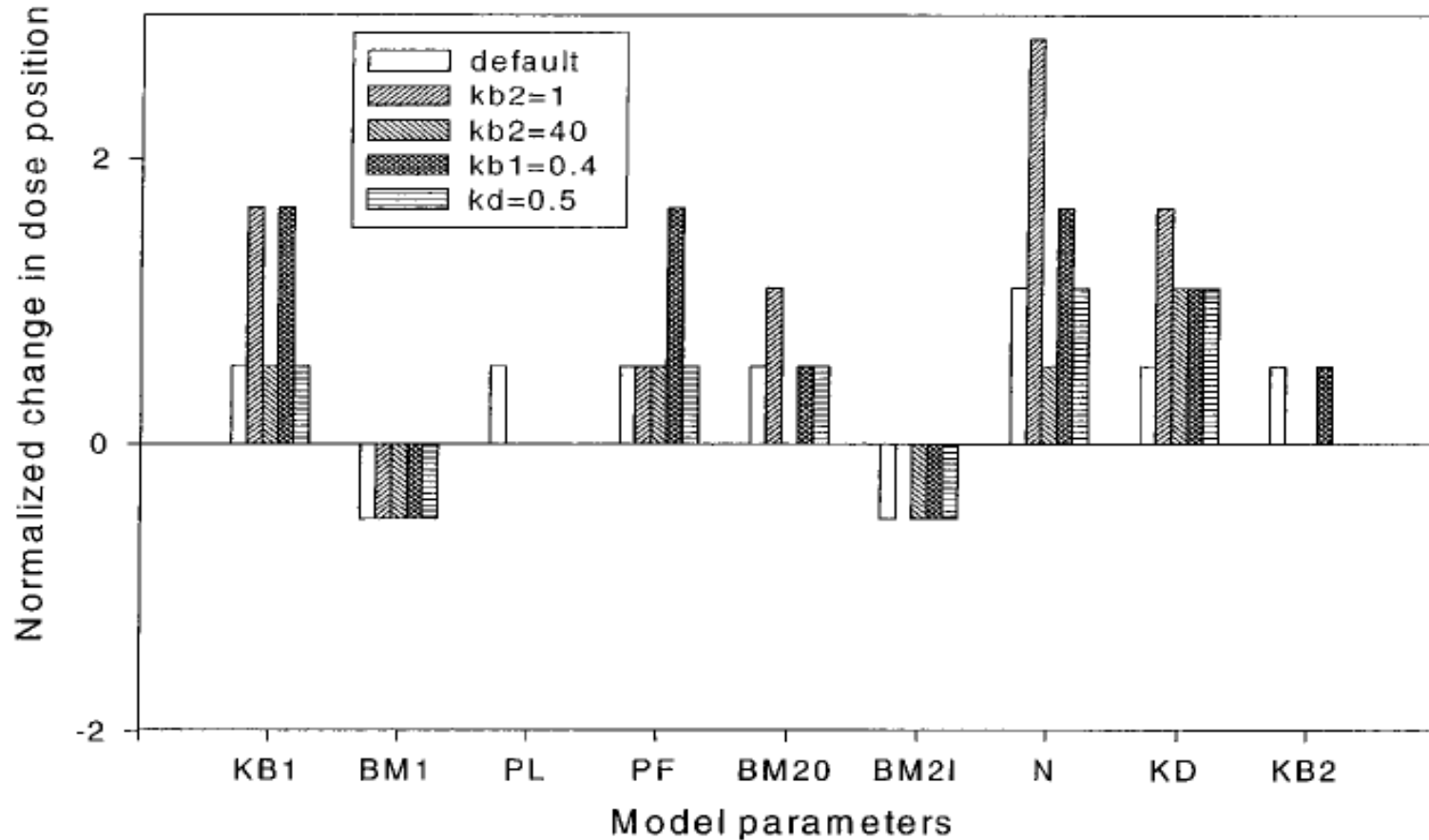


Liver concentration prediction after inhalation in rats– slopes are time dependent.





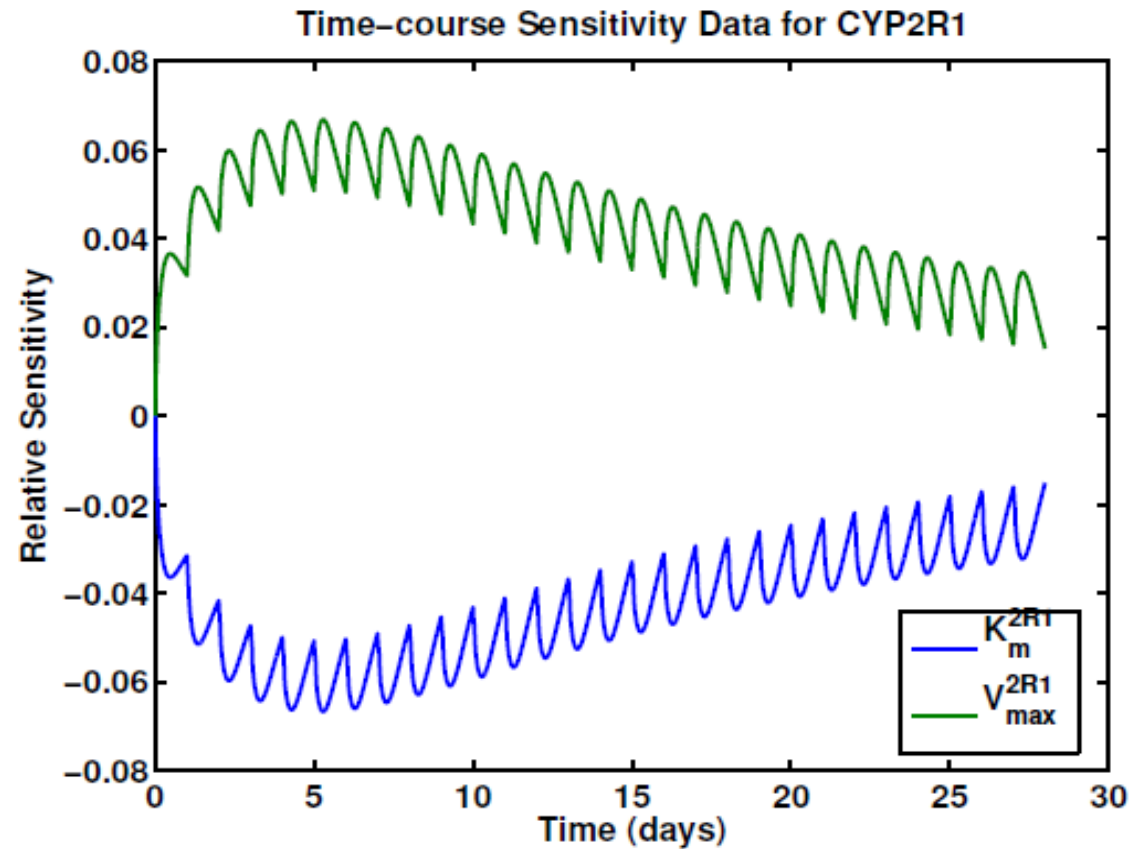
Normalized change in dose (measurement) for each model parameter at a fixed point in time.



KB1= Ah binding constant  
BM1 =Ah availability  
PL = liver PC  
PF = fat PC  
BM20 = CYP1A2 basal  
BM2I = CYP1A2 induction  
N = Hill coefficient  
KD =complex binding constant  
KB2 = CYP1A2 binding constant



Sensitivity coefficients add to zero-not uniquely identifiable for a human model.



(a)  $K_m^{2R1}$  and  $V_{max}^{2R1}$



# Mathematical tools used to rank Sensitivity Coefficients

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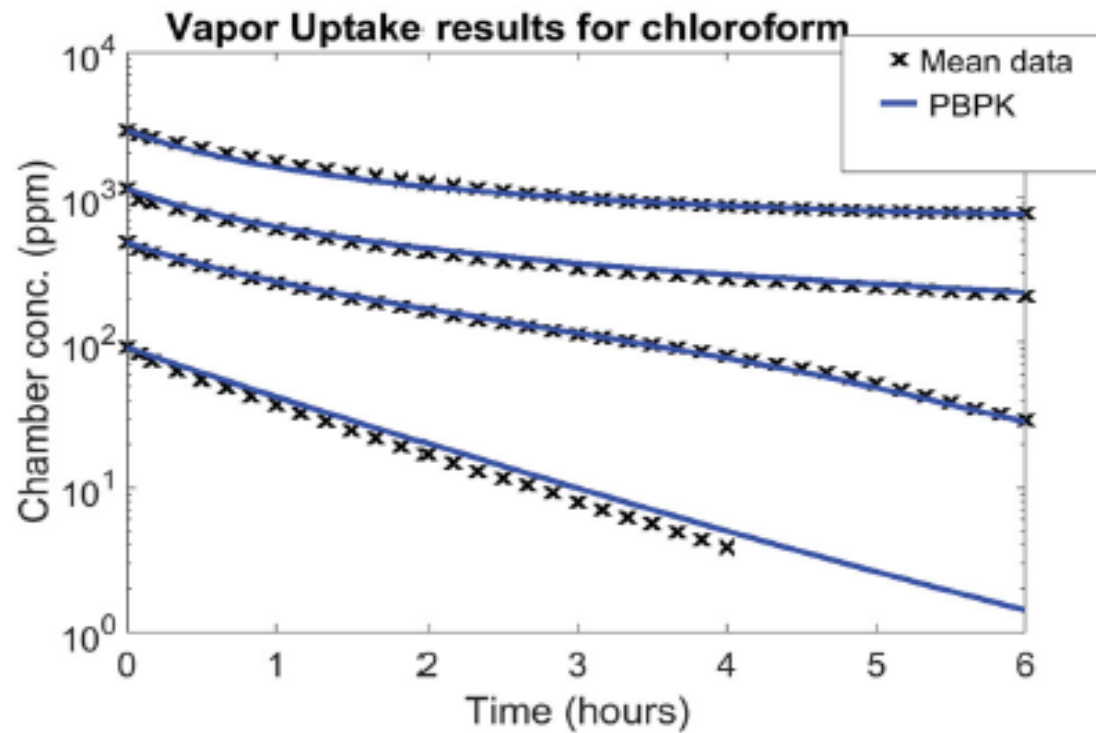
	Model Structure	Application
<i>Mathematical Techniques</i>	Linearization Taylor Series Expansion	Fisher Information Matrix Graphical or Visual Inspection Correlation matrix
<i>Model Complexity</i>	Difficult for more than 10 parameters and states	Can assess in models with more than 50 parameters and states

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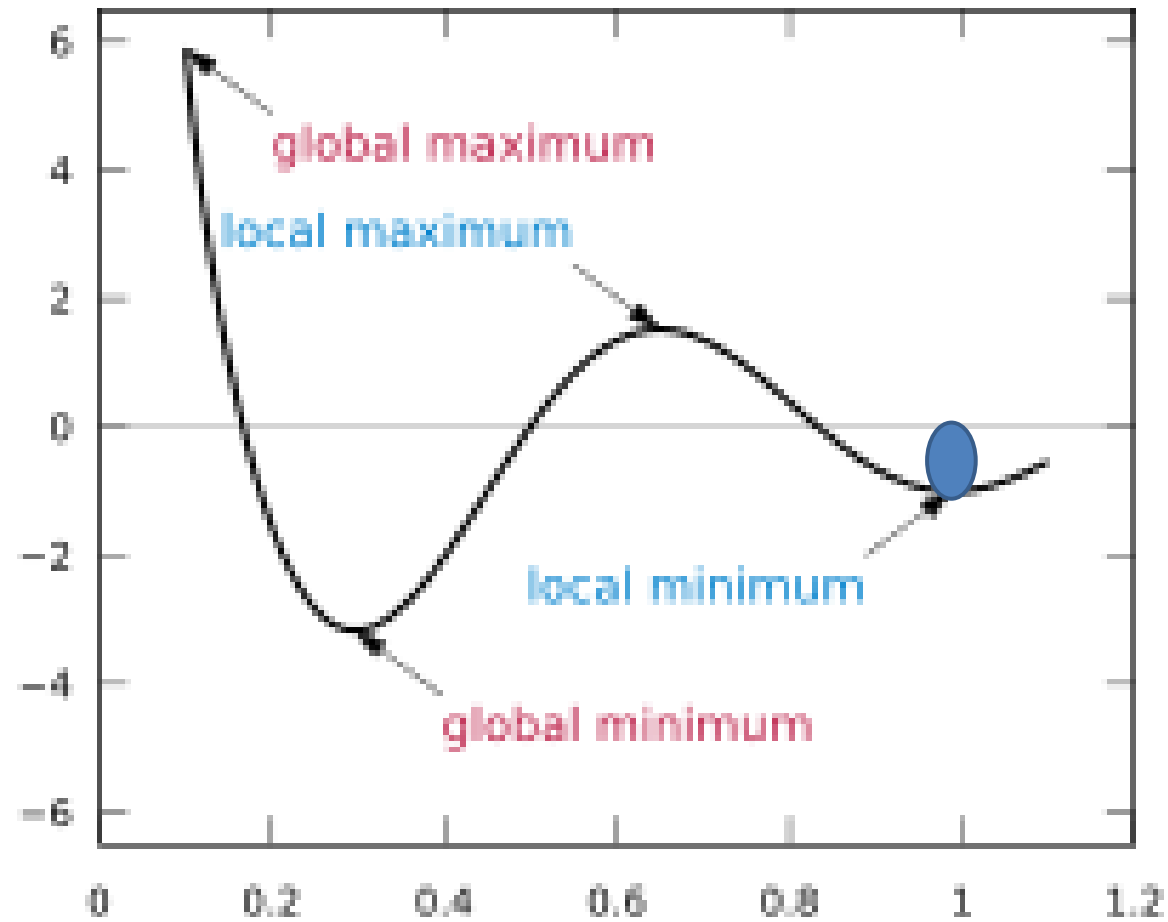
# How do we estimate unknowns?

- Optimization –
  - Take the difference between data and model simulation.
  - Make sure difference is as small as possible- Least Squares Sum (LSS).





# SA and global optimization

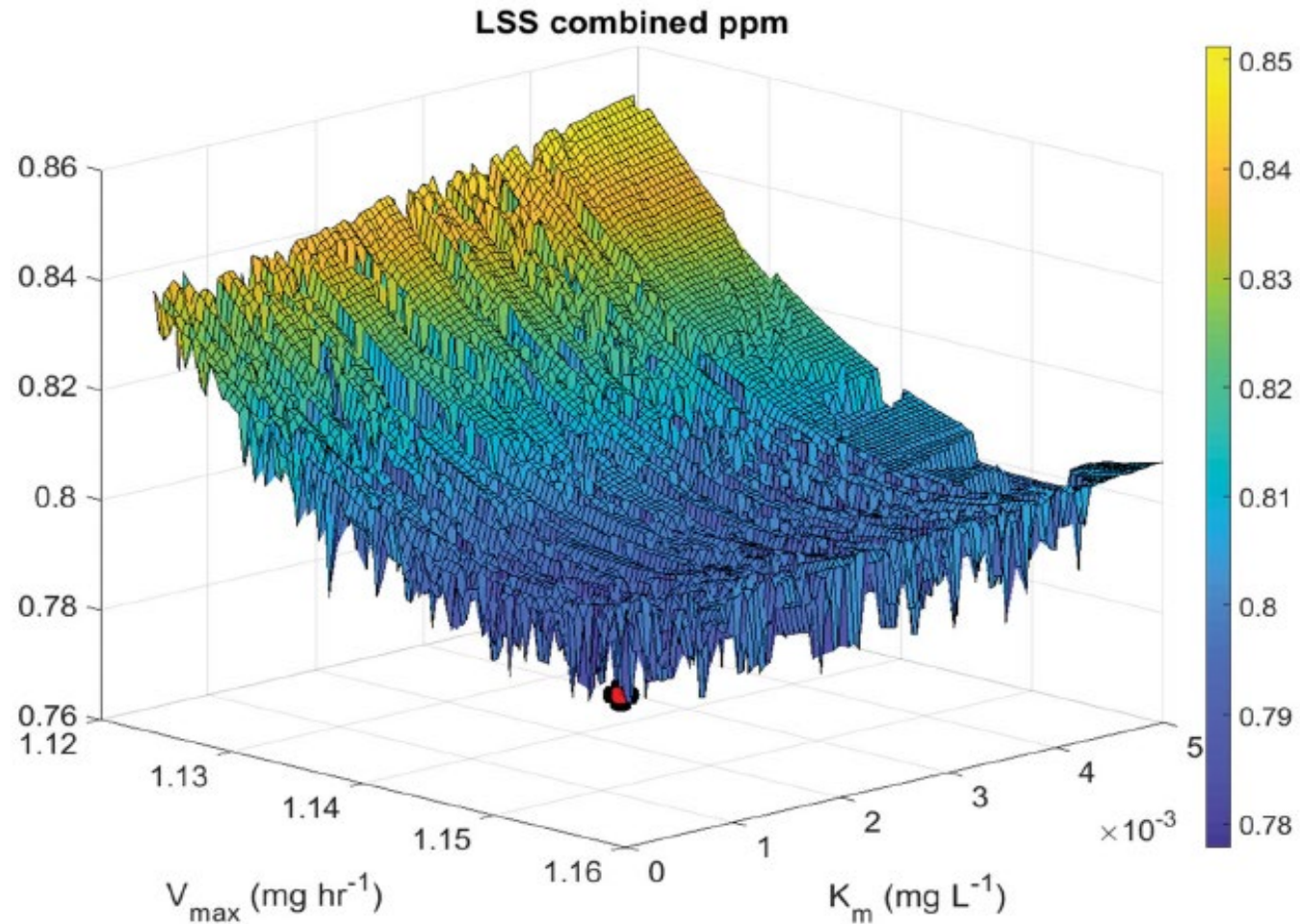


Use a global optimization algorithm to find true global minimum



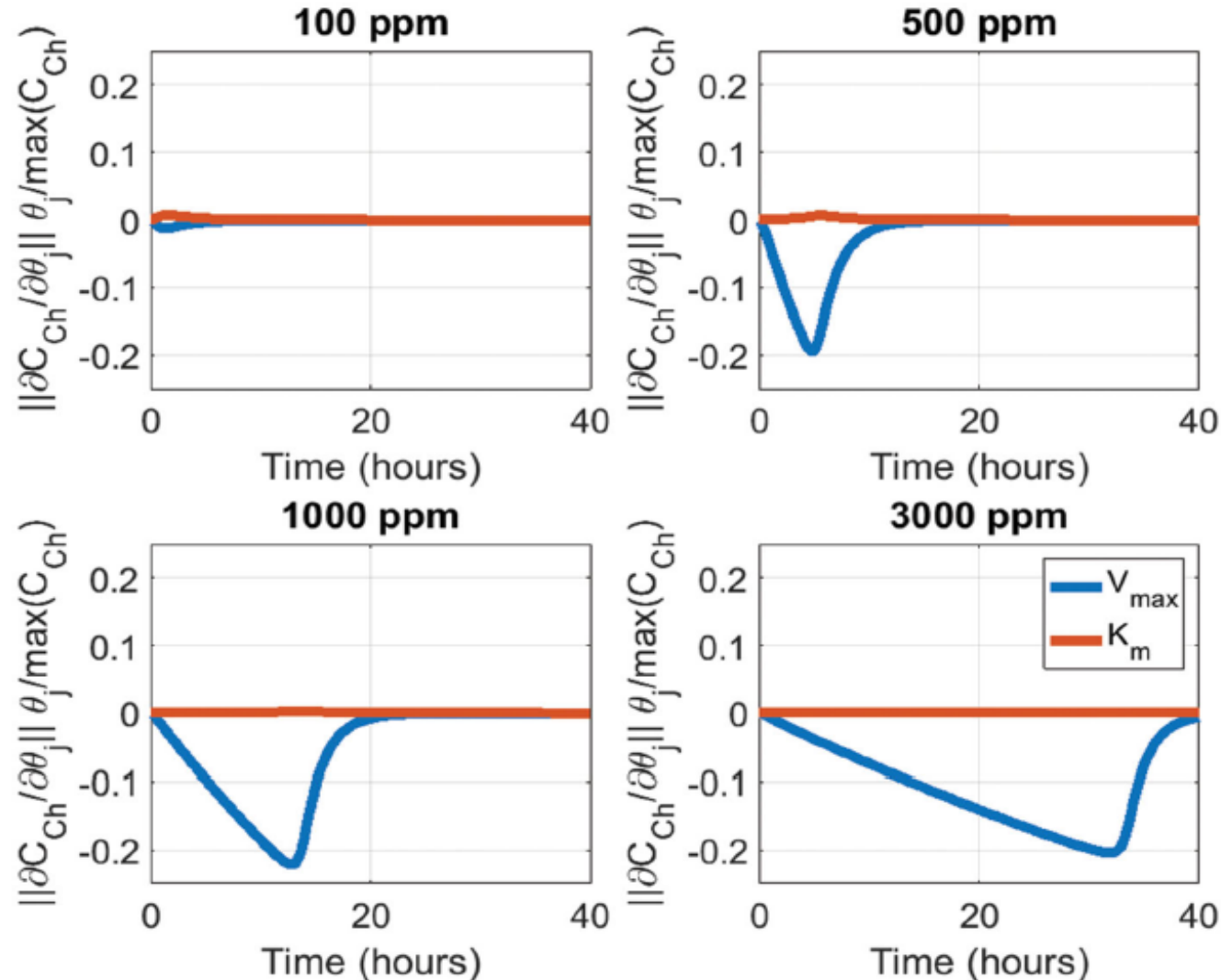


PBPK rat optimization surface with multiple peaks and valleys. Red dot is  $V_{\max}$  and  $K_m$  values.



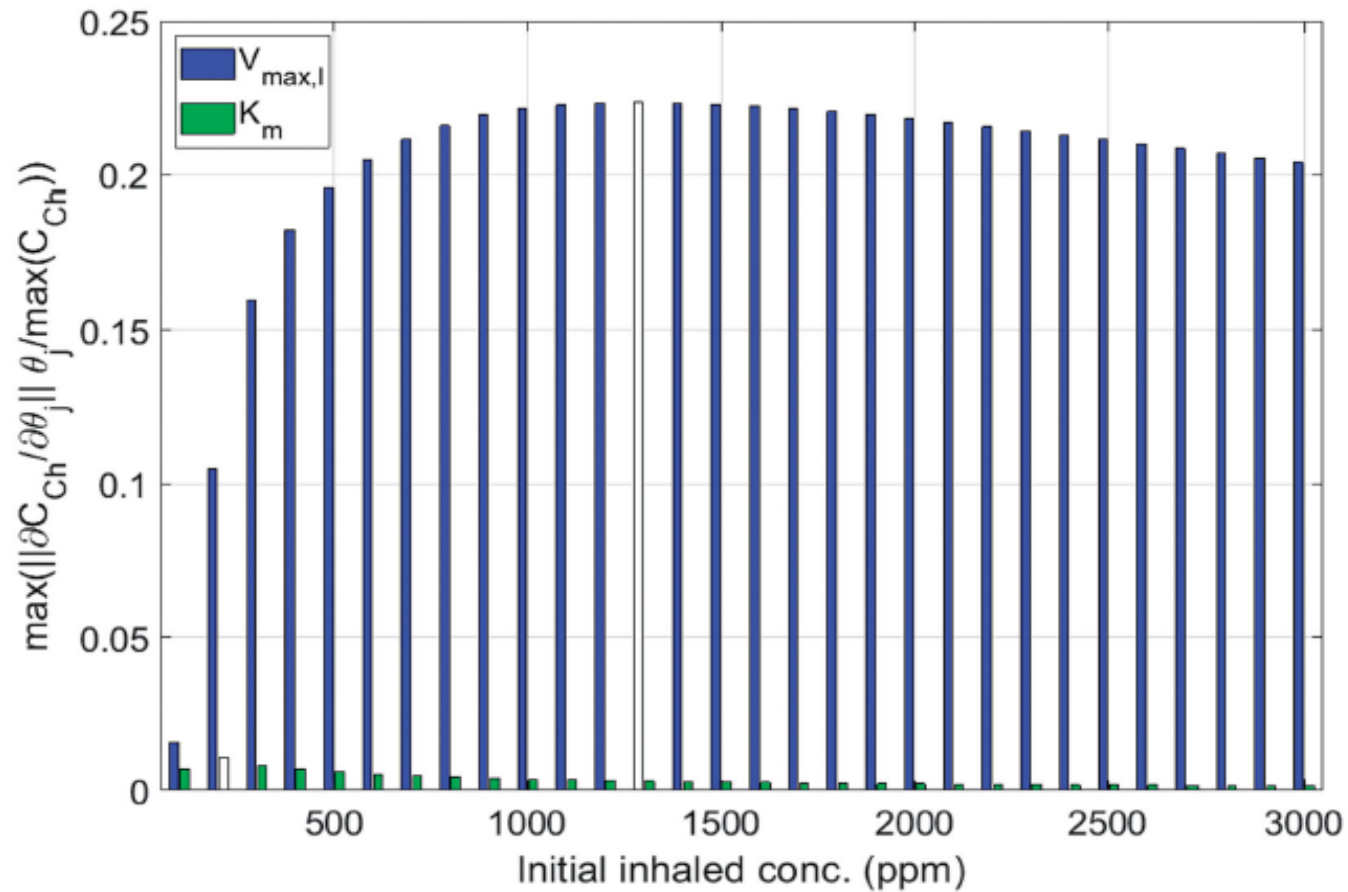


# Normalized SC for different inhalation concentrations in rat experiments for previous surface.





Normalized SC for given exposure can predict experimental concentrations- e.g. 1300 ppm for V<sub>max</sub> and 200 ppm for K<sub>m</sub>.





## How is SA performed?

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- Calculate Sensitivity Coefficients with Chain Rule and gradients (slopes) for each parameter of interest.
- Perform the calculations for each time point.
- Largest normalized sensitivity coefficients suggest time for estimable parameters.
- Ranking of most important parameters in model.
- Unique values will be obtained if SC do not cancel each other.



Thank You!

